Mathematical Modelling of Flow Distribution between the Danube Delta Branches

(Source: Danube Delta Hydrology (edited by V.N. Mikhailov). M.: GEOS, 2004. 448 p.***)***

The computer-based mathematical model for estimating the distribution and redistribution of water flows among the Danube Delta branches was developed by the expert team from the Faculty of Geography (Moscow State University), led by Dr. V.N. Mikhailov.

This mathematical model is based on the method that employs common modules of hydraulic resistance (Mikhailov et al., 1973; 1986 [3,4]). The advantage of this method stems from the fact that it enables the direct (without iteration procedure) analytical calculation of flow distribution between the delta branches with the use of specified, actual or designed, morphometric characteristics of deltaic watercourses: the length L, width *B,* average depth h, and roughness ratio *n.* This method has already been used for various deltas, including the Danube Delta (Polonsky et al., 1992[5]; Aleshkin, Kornilov, 2001[1]).

The method is based on the assumption that there is a full balance of water flows within a system of deltaic watercourses (i.e. there are no linear losses or inflows of water within a deltaic channel network) and the balance of a sum of water level decreases within a delta in either direction from the common bifurcation points of watercourses to the points of their inflow into a receiving water body.

The core of this method is the substitution of hydraulic resistance modules by a certain "common module", which is calculated using the electric circuit model with a delta system being a system of consecutive, parallel and consecutive/parallel connections between watercourses and their parts.

In this method, the distribution of flow discharges among the delta branches is calculated on the basis of the following formula:

$$Q_i / Q_j = \sqrt{F_{totalj} / F_{totali}}\ , \qquad (1)$$

where Q_i and Q_j are flow discharges at the sources of two adjacent channel systems *i* and *j* with a common bifurcation point, draining into the same coastal waters, $F_{total\,i}$ and $F_{total\,j}$ are common hydraulic resistance modules of these channel systems. These are calculated using the following formula:

$$F_{totali} = F_i + F_{total} = F_i + \frac{1}{\left(1/\sqrt{F_{left}} + 1/\sqrt{F_{right}}\right)^2}\ , \qquad (2)$$

where F_i is the resistance module of the main branch of a system under consideration, and F_{left} and F_{right} are common resistance modules for the left-side and right-side channel subsystems that are parts of system *i* (they are also calculated using formula (2)).

The common resistance module for a branch with a large mouth bar is calculated using the following formula

$$F_{total} = F_i + F_{bar} \qquad (3)$$

For a simple two-branch system, consisting of branches I and II, formula (1) has the following simple form:

$$Q_{\mathrm{I}} / Q_{//} = \sqrt{F_{\mathrm{II}} / F_{\mathrm{I}}} \qquad (4)$$

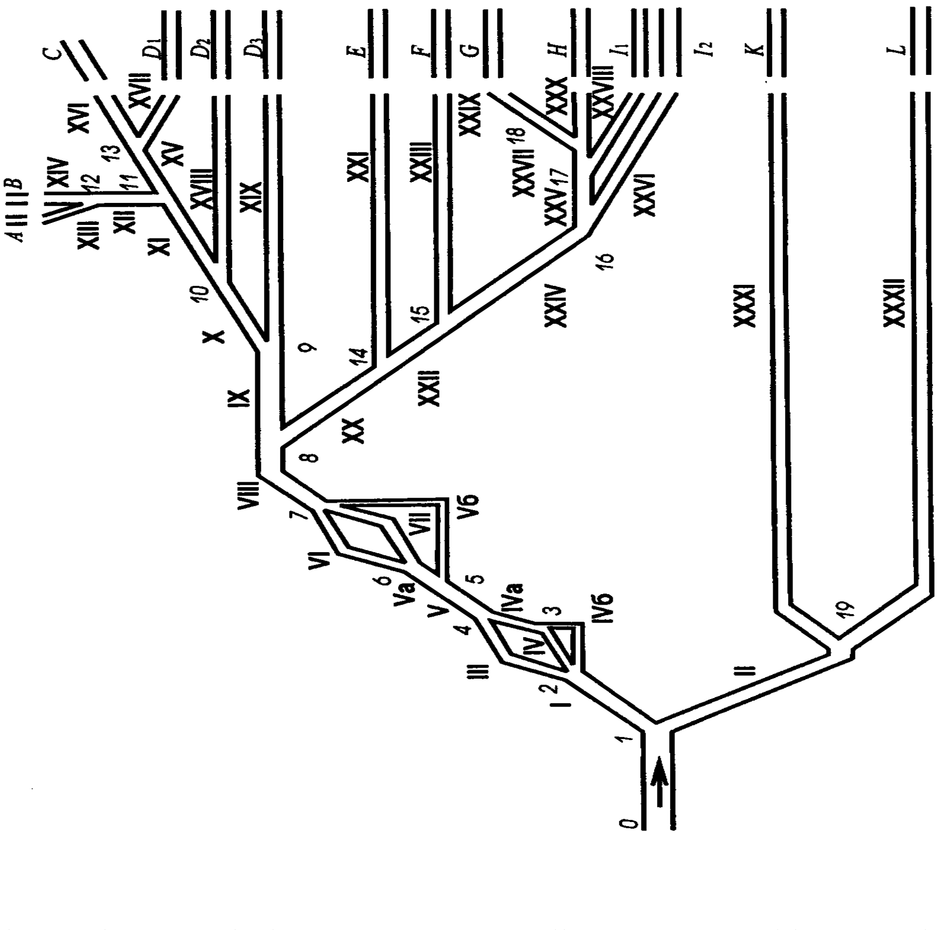
By N.N. Pavlovsky, the hydraulic resistance module of a branch and its part (including the mouth bar) is derived from the Chezy-Manning formula:

$$F = \Delta z / Q^2 = Ln^2 / B^2 h^{10/3} \tag{5}$$

where Δz is a decrease in water level within a section under consideration.

To enable the application of this model, the channel network of the Danube Delta was schematised (see Figure below). Roman numerals were used to assign an individual number to each branch and modelled section (i = I, II, III,..., XXXII). As can be seen from the Figure below, the present (test) modelling stage comprises 32 branches and their sections. The “zero” number was assigned to the Danube itself upstream from the delta head. Bifurcation points and junctions were marked with the Arabic numerals (*k = 1, 2, 3 ... 17);* 19 in total. The mouth bars were considered separately: they were marked with the Latin capitals *(j = A, B, C ... L),* 11 in total.

Figure – Schematic Layout of the Danube Delta Channel Network for Mathematical Modelling Purposes



This schematic layout does not take into account very small watercourses, either natural or artificial, that do not play any significant role in terms of flow distribution within the delta. Also, in order to keep the scheme simple, no account was taken of small flow diversions/exchanges between the channel systems. For example, the branches XVIII and XIX in reality merge as they reach the estuary, whereas the model scheme presents them as two watercourses draining into the sea separately. Where two or three watercourses reach the same mouth bar, the latter’s cross-section is conventionally divided into two or three equal parts (bars D_1, D_2, D_3; I_1, I_2).

The following approach was used to define the morphometric characteristics of mouth bars: bar width as a mean of a channel width in the mouth section and a bar crest length; water depth as a mean of average depths in the mouth section and along the bar crest.

In the first approximation, the same roughness ratio was used for all branches (n~0.023), according to (Mikhailov, 1971 [2]). It should be noted that, according to the formulae (1) – (5), there is no dependence between the flow distribution between the delta branches (Q) and the value of roughness ratio (n), assumed to be equal for various branches. In subsequent tests, the n value was used as the model's adjustment coefficient.

Special software was developed to enable the practical application of this mathematical model. The user interface is based on the Microsoft Excel XP software, which provides very convenient tools for information input and processing. The model software uses Visual Basic for Applications as a programming language.

The model was tested under different water regime phases of the Danube, for all currently known hydrological and morphometric characteristics of its branches. Testing results indicate that the model is fully capable of providing an adequate picture of flow distribution between the delta branches. The errors of estimates for certain branches under low-flow conditions (Q_{54} = 3000 m^3/s) do not exceed ±10%. The error margin appears to be slightly larger for high-flow conditions (Q_{54} = 10000 m^3/s).

The following conclusions were made with the help of this model:

1. The modelling results confirmed the significant impact of the St. George channel straightening (Table 1.2) on flow distribution between the major branches. The results of hydraulic calculation indicate that the reduced length of the St. George branch should have resulted in a 3% increase in flow received by this branch, and a further 2.2% increase in flow received by the Tulcea branch. Consequently, there should have been a 2.2% decrease in flow received by the Chilia branch.

2. **Special modelling exercise was conducted to analyse the potential hydrological consequences of the Bystre branch deepening for navigation. The results indicate that the deepening of the bar in the Bystre branch is likely to have a very minor impact on flow distribution within the Chilia delta. Under low-flow conditions, the flow discharged through the Bystre branch will increase only by 1-2 m^3/s even after the bar deepening to 7 and 9 m. The bar deepening is not likely to have any impact on the flow discharged through other branches.**

3. According to the estimates, the 1 km extension of jetties in the Sulina branch (with respective increase in its length) will result in a very minor (up to 3 m^3/s under low-flow conditions) decrease in flow discharged through this branch. The estimated increase in flow discharged through the adjacent St. George branch under low-flow conditions is only 1 m^3/s.

4. More significant impact on flow distribution within the entire delta is likely to be associated with the progressive water erosion in the St. George branch, caused by a significant reduction in its length. The estimates indicate that an additional 0.2 m increase in water depths due to the St. George branch channel degradation will result in an increase of flow discharged through this branch by 40 m^3/s under low-flow conditions, which is 1.3% of the total Danube flow under low-flow conditions (Q_{54} = 3000 m^3/s). This will further promote the current pattern of flow redistribution, dominated by the Tulcea system.

5. Certain change in flow distribution may be likely as a result of rift deepening (by 1-3 m) in some sections of the Chilia branch as part of the Danube-Black Sea Navigation Route Project.

The completed hydraulic calculation procedure involved the route dividing into the sections that are planned to be deepened and those that are not. The initial modelling results indicate that with 12 rifts deepened in the Chilia branch between the Ismail Chatal and the Chilia Delta head, the flow discharged through the Chilia Branch will increase by 24 m^3/s (0.80% of the Danube flow) under low-flow conditions (Q_{54}= 3000 m^3/s).

6. The estimated 0.2 m increase in depths in the St. George branch (due to the progressive channel degradation) will counteract with the deepening of rifts in the Chilia branch, and the effect of channel degradation in the St. George branch on the flow distribution pattern in the entire Danube Delta is expected to be stronger. Under low-flow conditions (Q_{54}=3000 m^3/s), flow discharged through the St. George branch will increase by 25 m^3/s (0.83% of the Danube flow) relative to the current discharges, whereas the flow received by the adjacent Sulina branch will decrease by 22 m^3/s (0.73% of the Danube flow). Consequently, the flow discharged through the Tulcea branch will increase by 3 m^3/s (0.10% of the Danube flow), with the proportional decrease in flow received by the Chilia branch. From this, the artificial deepening of the Chilia branch is not likely to alter the general trend of flow redistribution to the benefit of the Tulcea and St. George branches, but only slow down its progress.

When assessing the impact of proposed navigation route on the flow distribution between the Danube Delta branches, one should take into account the prediction of the ***natural development*** of the Chilia delta and its branches, which indicates that, despite an overall decrease in flow availability in the Chilia branch, the flow received by the Starostambulske branch will remain relatively stable. The Bystre branch will continue to be the most dynamic part of the Chilia delta. By 2015, it is expected to account for 19.2% of the Danube flow, and flow discharges in the Bystre branch will be higher than those in the Starostambulske branch downstream of bifurcation point.

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